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THE STANDARD MODEL PREDICTION FOR ε'/ε

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We briefly review the most important ingredients of a new Standard Model analysis of ε'/ε which takes into account the strong enhancement induced by final state interactions.

1 Introduction

The study of non–leptonic $K \to \pi\pi$ decays is of great importance in the understanding of CP violation mechanisms within the Standard Model and beyond. In particular, a crucial quantity is the parameter ε'/ε which measures the magnitude of the direct CP violation in the Kaon system. The experimental situation has been greatly improved recently, after the measurement by NA48 at CERN and KTeV at Fermilab. The new quoted experimental world average[1] is $\mathrm{Re}\,(\varepsilon'/\varepsilon) = (19.3 \pm 2.4)\cdot 10^{-4}$, providing a clear evidence of the existence of direct CP violation with a non–zero value of ε'/ε.

The theoretical prediction of ε'/ε still suffers from many uncertainties which mainly affect the determinaton of the long–distance contributions to $K \to \pi\pi$ matrix elements and the matching with the short–distance part. Recently, it has been observed[2,3] that the soft final state interactions (FSI) of the two pions play an important role in the determination of ε'/ε. From the measured π–π phase shifts one can easily infer that FSI generate a strong enhancement of the predicted value of ε'/ε by roughly a factor of two[2,3], providing a good agreement with the experimental value.

Here, we discuss a few basic aspects of a new Standard Model evaluation of ε'/ε which has been proposed in Refs.[2,3,4] and includes FSI effects. In addition to the large infrared logarithms generated by FSI there are the well known large ultraviolet logarithms that govern the short–distance evolution of the Wilson coefficients. Both these logarithms need to be resummed and included in the evaluation of ε'/ε. The large–N_C expansion[6,7] provides a convenient framework, with a well defined power counting, to properly include all these corrections.

In Sec. 2 we review the calculation of long–distance $K \to \pi\pi$ matrix elements with the inclusion of FSI effects, while Sec. 3 is devoted to the evaluation of ε'/ε.

2 $K \to \pi\pi$ matrix elements

The long–distance realization of matrix elements among light pseudoscalar mesons can be obtained within Chiral Perturbation Theory (ChPT), as an expansion in powers of momenta and light quark masses[3]. The $K \to \pi\pi$ amplitudes with $I = 0, 2$ generated by the lowest–order ChPT lagrangian are

$$
\begin{aligned}
A_0 &= -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sqrt{2} f \\
&\left\{ \left(g_8 + \frac{1}{9}\, g_{27} \right) (M_K - M_\pi^2) - \frac{2}{3} f^2 e^2 g_{EM} \right\}, \\
A_2 &= -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \,\frac{2}{9} f \,\left\{ 5\, g_{27}\, (M_K - M_\pi^2) \right. \\
&\left. -3 f^2 e^2 g_{EM} \right\}, \qquad (1)
\end{aligned}
$$

where g_8, g_{27} and g_{EM} are the chiral couplings and the isospin decomposition of Ref.[3] has been used.

FSI start at next–to–leading order in the chiral expansion. To resum those effects the Omnès approach for $K \to \pi\pi$ decays has been proposed in Ref.[2] and discussed in detail in Ref.[3]. For CP–conserving amplitudes, where the $e^2 g_{EM}$ corrections can be safely neglected, the most general Omnès solution for the on–shell amplitude can be written as follows

$$\mathcal{A}_I = \left(M_K^2 - M_\pi^2\right)\, \Omega_I(M_K^2, s_0)\, a_I(s_0) \qquad (2)$$
$$= \left(M_K^2 - M_\pi^2\right)\, \Re_I(M_K^2, s_0)\, a_I(s_0)\, e^{i\delta_0^I(M_K^2)}\,.$$

The Omnès factor $\Omega_I(M_K^2, s_0)$ provides an evolution of the amplitude from low energy values (the subtraction point s_0), where the ChPT momentum expansion can be trusted, to higher energy values, through the exponentiation of the infrared effects due to FSI. It can be split into the dispersive contribution $\Re_I(M_K^2, s_0)$ and the phase shift exponential[a]. Taking a low subtraction point $s_0 = 0$, we have shown[3] that one can just multiply the tree–level formulae (1) with the experimentally determined Omnès exponentials [3]. The two dispersive correction factors thus obtained [3] are $\Re_0(M_K^2, 0) = 1.55 \pm 0.10$ and $\Re_2(M_K^2, 0) = 0.92 \pm 0.03$.

The complete derivation of the Omnès solution for $K \to \pi\pi$ amplitudes makes use of Time–Reversal invariance, so that it can be strictly applied only to CP–conserving amplitudes. However, working at the first order in the Fermi coupling, the CP–odd phase is fully contained in the ratio of CKM matrix elements $\tau = V_{td}\, V_{ts}^* / V_{ud}\, V_{us}^*$ which multiplies the short–distance Wilson coefficients. Thus, decomposing the isospin amplitude as $\mathcal{A}_I = \mathcal{A}_I^{CP} + \tau\, \mathcal{A}_I^{\not{C}\not{P}}$, the Omnès solution can be derived for the two amplitudes $\mathcal{A}_I^{CP}$ and $\mathcal{A}_I^{\not{C}\not{P}}$ which respect Time–Reversal invariance. In a more standard notation, $\mathrm{Re}A_I \approx \mathcal{A}_I^{CP}$ and $\mathrm{Im}A_I = \mathrm{Im}(\tau)\, \mathcal{A}_I^{\not{C}\not{P}}$, where the absorptive phases have been already factored out through $\mathcal{A}_I = A_I\, e^{i\delta_0^I}$.

[a]For the electroweak penguin operator Q_8, the lowest order chiral contribution is a constant proportional to $e^2 g_{EM}$, where the explicit $SU(3)$ breaking is induced by the quark charge matrix, instead of the term $M_K^2 - M_\pi^2$.

3 The parameter ε'/ε

The direct CP violation parameter ε'/ε can be written in terms of the definite isospin $K \to \pi\pi$ amplitudes as follows

$$\frac{\varepsilon'}{\varepsilon} = e^{i\Phi}\, \frac{\omega}{\sqrt{2}|\epsilon|} \left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \right], \qquad (3)$$

where the phase $\Phi = \Phi_{\varepsilon'} - \Phi_\varepsilon \simeq 0$ and $\omega = \mathrm{Re}A_2/\mathrm{Re}A_0$. Since the hadronic matrix elements are quite uncertain theoretically, the CP–conserving amplitudes $\mathrm{Re}A_I$, and thus the factor ω, are usually set to their experimentally determined values; this automatically includes the FSI effect. All the rest in the numerator has been *theoretically* predicted mostly via short–distance calculations, which therefore do not include FSI corrections. This produces a mismatch which can be easily corrected by introducing in the numerator the appropriate dispersive factors $\Re_I$ for FSI effects. The evaluation of ε'/ε proposed in Ref.[4] proceeds through the following steps:

- All short–distance Wilson coefficients are evolved at next-to-leading logarithmic order[10,11] down to the charm quark mass scale $\mu = m_c$. All gluonic corrections of $\mathcal{O}(\alpha_s^n t^n)$ and $\mathcal{O}(\alpha_s^{n+1} t^n)$ are already known. Moreover, the full m_t/M_W dependence (at lowest order in α_s) has been taken into account. This provides the resummation of the large ultraviolet logarithms $t \equiv \ln(M/m)$, where M and m refer to any scales appearing in the evolution from M_W down to m_c.
- At the scale $\mu \sim 1$ GeV the $1/N_C$ expansion can be safely implemented. At this scale the logarithms which govern the evolution of the Wilson coefficients remain small

$\sim \ln{(m_c/\mu)}$ so that the $1/N_C$ expansion has a clear meaning within the usual perturbative expansion in powers of α_s. In the large-N_C limit both the Wilson coefficients $C_i(\mu)$ and the long–distance matrix elements $\langle Q_i(\mu)\rangle_I$ can be computed and the matching at the scale $\mu \le m_c$ can be done *exactly*.

• The Omnès procedure can be applied to the individual matrix elements $\langle Q_i\rangle_I$. Since the FSI effect is next–to–leading in the $1/N_C$ expansion one can include it via the realization $\langle Q_i(\mu)\rangle_I \sim \langle Q_i(\mu)\rangle_I^{N_C\to\infty} \times \Re_I$, while avoiding any double counting.

The large-N_C realization of the matrix elements $\langle Q_i(\mu)\rangle_I$, with $i \neq 6,8$ is always a product of the matrix elements of colour–singlet vector or axial currents. Each of them being an observable, the corresponding matrix element is renormalization scale and scheme independent. The same is true for the corresponding Wilson coefficients in the large–N_C limit, so that the matching is exact. The large–N_C realization of $\langle Q_{6(8)}(\mu)\rangle_I$ scales like the inverse of the squared fermion mass, being the product of colour–singlet scalar and pseudoscalar currents. Conversely, the Wilson coefficients of the operators Q_6 and Q_8 scale proportionally to the square of a quark mass in the large–N_c limit, so that again the matching is exact.

The connection between the tree level ChPT amplitudes (1) and the large–N_C realization of the operators Q_i can be clarified as follows. At the lowest non trivial order in the chiral expansion, the large–N_C realization of an operator Q_i gives the contribution of Q_i itself to the chiral couplings g_8, or g_{27} or g_{EM} (according to its transformation properties) in the large–N_C limit. In this sense, the Omnès solution formulated in Sec. 2 can be applied directly to the large–N_C matrix elements of the operators Q_i with the dispersive factors $\Re_I(M_K^2, 0)$ already estimated.

A preliminary Standard Model analysis of ε'/ε gives $\varepsilon'/\varepsilon = (17 \pm 6)\cdot 10^{-4}$, where the error is dominated by the $1/N_C$ approximation. Further refinement and details of the analysis will be given elsewhere[4].

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